The displacement theorem, and the aspect no-go

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Abstract

Two small results, stated precisely and machine-checked (22/22 pre-registered predictions across the project): a pigeonhole-style displacement theorem — the metalanguage, not the primitive, is what cannot be eliminated — and a two-line model-theoretic no-go for aspect. Both are plainly labeled where they are trivial.

The two residues from the 2026-07-26 adjudication, now discharged. Machine-checked in `experiments/exp_d_displacement.py` *(4/4) and* `experiments/exp_e_aspect_nogo.py` *(4/4). Total across the project: 22/22 pre-registered predictions.*

Both results are small. Neither is deep mathematics — the first is a pigeonhole argument and the second is a two-line model-theoretic observation. They are written down because the prior-art sweep found neither stated anywhere, because together they close Thread B properly rather than leaving it dangling, and because stating a modest thing precisely is worth more than gesturing at a grand one. We say plainly below where each is trivial.

Part I — The Displacement Theorem

Motivation

Thread B conjectured that *the metalanguage, not the primitive, is what cannot be eliminated.* The literature has the special cases: Tarski's undefinability theorem for the truth predicate, Kripke's fixed points and their "ghost of the hierarchy," Varela's re-entry. What it does not have is the general statement about distinction specifically — the claim that killing distinction in the object language forces it into the metalanguage.

Definitions

Definition 1 (specification system). A *specification system* is a triple $(\mathcal{E}, \mathcal{S}, \delta)$ where $\mathcal{E}$ is a set of expressions, $\mathcal{S}$ a set of structures, and $\delta : \mathcal{E} \to \mathcal{S}$ an interpretation.

Definition 2 (discriminating). $(\mathcal{E}, \mathcal{S}, \delta)$ is *discriminating for a class* $\mathcal{C} \subseteq \mathcal{S}$ iff every $S \in \mathcal{C}$ has some $e \in \mathcal{E}$ with $\delta(e) = S$.

Definition 3 (distinction). A set X *contains a distinction* iff $|X| \geq 2$. The object level of a structure is *distinction-free* iff its domain is a singleton.

The reading: $\mathcal{C}$ is the class of alternatives an assertion is meant to choose among. To assert "the world is distinction-free" is to select one member of a $\mathcal{C}$ that contains at least one other member — otherwise the assertion excludes nothing and says nothing.

The theorem

> Theorem (Displacement). Let $(\mathcal{E}, \mathcal{S}, \delta)$ be discriminating for $\mathcal{C}$. Then $|\mathcal{E}| \geq |\mathcal{C}|$. In particular, if $|\mathcal{C}| \geq 2$ then $\mathcal{E}$ contains a distinction.

Proof. Discrimination says δ restricted to $\delta^{-1}(\mathcal{C})$ is onto $\mathcal{C}$. A surjection from a subset of $\mathcal{E}$ onto $\mathcal{C}$ forces $|\mathcal{E}| \geq |\mathcal{C}|$. □

That is the whole proof. It is pigeonhole, and we are not going to dress it up.

> Corollary 1 (Displacement proper). A structure may have a distinction-free object level, but it can be *specified as such*, as against alternatives, only from within a metalanguage that itself contains a distinction.

Proof. The object level being a singleton is consistent (verified: check F2). Specifying it as against any alternative instantiates the theorem with $|\mathcal{C}| \geq 2$, giving $|\mathcal{E}| \geq 2$. □

> Corollary 2 (Conservation). Eliminating object-level distinction does not reduce the distinction required to specify the result. It relocates it, and the amount relocated grows with the number of alternatives excluded: $|\mathcal{E}| \geq |\mathcal{C}|$, i.e. at least $\log_2 |\mathcal{C}|$ bits of metalinguistic distinction.

> Corollary 3 (No self-exemption). The displacement cannot be escaped by letting the metalanguage be its own object language. A self-describing, distinction-free language denotes at most one thing, hence cannot distinguish itself from any alternative — including from a language that does contain distinctions.

Proof. Set $\mathcal{E} = \mathcal{S}$ with $|\mathcal{E}| = 1$ and apply the theorem (verified: check F4). □

Machine verification

`experiments/exp_d_displacement.py`, 4/4 pre-registered:

Check	Content	Predicted	Got
F1	Two distinct structures denoted, one-expression metalanguage	UNSAT	UNSAT
F2	One-element object domain with a ≥2-expression metalanguage	SAT	SAT
F3	Counting: two expressions asked to denote three structures	UNSAT	UNSAT

Check	Content	Predicted	Got
F4	Self-describing distinction-free language denoting two things	UNSAT	UNSAT

F1 and F2 *together* are the displacement. Neither alone says anything: F2 shows the object level really can be emptied, F1 shows the bill is then paid in full one level up.

What this is, and is not

It is not new mathematics. It is the pigeonhole principle, and any competent reader will see the proof before finishing the statement.

What it does is locate three known things as one thing. It is Spencer-Brown's axiom — *no indication without distinction* — applied to the metalanguage rather than the object language. It is Shannon's observation that a single-symbol source has zero entropy, read as a constraint on ontology rather than on channels. It is the general form of the pattern Tarski proved for the truth predicate. And it is Bateson's "a difference that makes a difference," made into an inequality.

Its actual use is diagnostic, and it is what closes Thread B. The fork asked whether distinction or representation is more fundamental, and looked for the answer in the object-level ontology. The theorem says no object-level ontology can settle it, because every candidate ontology — including the distinction-free one — has to be *stated*, and the stating is where the distinction lives. That is why the Z3 fork checks came back "satisfiable both ways." It was never an object-level question.

Part II — The Aspect No-Go

Motivation

Thread B's Candidate 3 was the only escape from Hypothesis A that did not obviously beg the question: perhaps representation is not a relation between two things but *the possibility that the primitive is present in more than one way* — appearance prior to there being two entities. The tradition is respectable and entirely informal: Advaita's *vivarta-vāda*, Plotinus's undiminished emanation, Donald Baxter's aspects ("qualitative difference without numerical difference"), Chisholm's adverbialism. The sweep found no formal treatment. So: can it be formalized without smuggling in a two-element index set?

Definitions

Definition 4 (aspect structure). An *aspect structure* is $(P, W, \mathrm{App}, \approx)$ with P a set of primitives, W a set of ways of appearing, $\mathrm{App} \subseteq W \times P$ read "the primitive is present in this way," and $\approx$ an equivalence on W read as numerical identity (Baxter's move: aspects may be numerically identical yet qualitatively differ).

Three conditions:

- MANY: $\exists\, w_1 \neq w_2,\ \exists p$ with $\mathrm{App}(w_1, p)$ and $\mathrm{App}(w_2, p)$.
- ONE: $|P| = 1$.

- IoI (identity of indiscernibles for ways): if w_1 and w_2 bear App to exactly the same primitives, then $w_1 = w_2$.

The result

> Proposition (Aspect dilemma). MANY + ONE + $\approx$-collapse is satisfiable — the aspect position is formally consistent. But MANY stated in the $\approx$-quotient — that is, using the theory's *own* identity relation — is unsatisfiable.
>
> Theorem (Aspect no-go). MANY + ONE + IoI is unsatisfiable.

Proof. Under ONE there is a single primitive p_0, so the only App-fact available about any way w is the truth value of App(w, p_0). Take w_1, w_2 witnessing MANY: both satisfy App(w_i, p_0), hence they agree on every App-fact, hence they are indiscernible. IoI gives $w_1 = w_2$, contradicting MANY. □

> Corollary (the cost, located). MANY + ONE + IoI becomes satisfiable exactly when a predicate D on W is added that discriminates ways. That predicate is the smuggled index set: structure whose only function is to tell ways apart — a distinction under another name.

Machine verification

`experiments/exp_e_aspect_nogo.py`, 4/4 pre-registered:

Check	Content	Predicted	Got
G1	MANY + one primitive + numerical collapse	SAT	SAT
G2	MANY stated in the theory's own quotient	UNSAT	UNSAT
G3	MANY + one primitive + identity of indiscernibles	UNSAT	UNSAT
G4	G3 plus a discriminating predicate	SAT	SAT

The answer

No. "One primitive, many manners of presence" cannot be formalized without paying one of exactly two prices:

1. Reject the identity of indiscernibles for ways (G1 vs G3). This is available — G1 shows the position is consistent — but it means holding that two ways can differ while nothing whatsoever distinguishes them. And G2 shows what that costs: under the theory's *own* identity relation, there are not many ways after all. "Many manners of presence" comes out true only under a description the theory simultaneously declares not to mark a real difference.
2. Add a discriminating predicate (G4) — which is the index set the question asked us to avoid, reintroduced under another name.

So Candidate 3 does not evade Hypothesis A. It relocates the evasion into a currency it declines to count — which is, precisely, the Displacement Theorem showing up again one level down. The two results are the same phenomenon: kill a distinction and it reappears in whatever you used to kill it.

Status of Thread B after these two results

Closed, and closed cleanly rather than abandoned.

- The fork (distinction-first vs representation-first) is definitional, not factual — settled by whether self-representation counts (exp B, 5/5), and answered by Peirce a century ago with the order-of-being / order-of-knowing distinction.
- The metalanguage conjecture was right, and now has a theorem and a proof: distinction is displaced, not eliminated, with a counting bound.
- The appearance escape is closed by a no-go with its cost located.

Nothing here rehabilitates Ontogenic Mathematics as a foundations program. What it does is leave the strand in a state where someone can pick it up and know exactly what is settled, what it cost to settle, and that there is no fourth move waiting at the object level.

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